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RESEARCH MEMORANDUM

SOME CONSIDERATIONS OF THE INFLUENCE OF BODY CROSS-SECTIONAL SHAPE ON THE LIFTING EFFICIENCY OF WING-BODY COMBINATIONS AT SUPERSONIC SPEEDS

By E. B. Klunker and Keith C. Harder

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

WASHINGTON

October 19, 1956

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SUMMARY

Linearized theory is used to estimate the order of magnitude of the beneficial effects that can be obtained by shaping the body of a wing-body configuration to generate lift indirectly at supersonic speeds. The analysis applies to a class of wing-body configurations which is believed to exhibit the essential features of this type of interference; the body is cylindrical upstream of a wing which has supersonic edges and the parts of the body above and below the wing are each semicircular and of different radii. Calculations are presented for a Mach number of $\sqrt{2}$ for a particular family of bodies and a sonic-edge delta wing.

INTRODUCTION

At supersonic speeds, lift can be generated by locating a wing in the interference field of another component so that expansion waves occur over the upper surface of the wing and compression waves over the lower surface. Recent investigations have shown that in some cases efficient lifting systems can be obtained by utilizing this interference. For example, in reference 1, Rossow considered the interference effects between a wing and planar elements which simulate a body or represent vanes at the wing tips. Several combinations were determined which, according to linear theory, generate lift more efficiently than the wing alone. However, as pointed out by Rossow, the friction drag of the fins may completely offset the gain achieved.

Aside from a wing-cone system analyzed by Rossow and a very limited amount of experimental work concerned with some effects of body cross-sectional shape, the studies dealing with the generation of indirect lift have been concerned primarily with the pressure fields and forces developed by interfering planar elements. Although the results of such studies are suggestive of the interference effects between wing and bodies, they do not apply directly to wing-body combinations wherein the body encloses a certain volume and supports part of the lift. The purpose of the present investigation is to obtain some idea of the magnitude of the beneficial effects that can be obtained by shaping the body of a wing-body configuration to generate lift indirectly. Accordingly, a configuration was selected for study which is believed to exhibit the essential features of this type of interference and also yields to a relatively simple analysis. This configuration is composed of a wing with supersonic edges and a body which is cylindrical upstream of the wing, the parts of the body above and below the wing each being semicircular and of different radii. Calculations are presented for a Mach number of $\sqrt{2}$ for a particular family of body shapes and a sonic-edge delta wing.

SYMBOLS

$A(x)$	area distribution in upper half-space
$\Delta A(x) = A(x) - \frac{\pi R_0^2}{2}$	
b	nondimensional wing span
B	constant
c	nondimensional wing chord
C_L	lift coefficient based on total wing area
C_D	drag coefficient based on total wing area
D	drag
$f(x)$	body source strength
$g_{2n}(x)$	function defined in equation (6)
$h_n(x)$	multipole strength
L	lift
M	free-stream Mach number
p	pressure
q	free-stream dynamic pressure

r	nondimensional radius in cylindrical coordinates
R_0	radius of cylinder
r_b	nondimensional body radius
U	free-stream velocity
W_{2n}	tabulated function
x,y,z	nondimensional Cartesian coordinate system
α	wing angle of attack

$\beta = \sqrt{M^2 - 1}$

Φ	velocity potential
ϕ	disturbance velocity potential
θ	azimuth angle in cylindrical coordinates

Subscripts:

b	refers to body potential
i	refers to interference potential
w	refers to wing potential
∞	refers to free-stream conditions

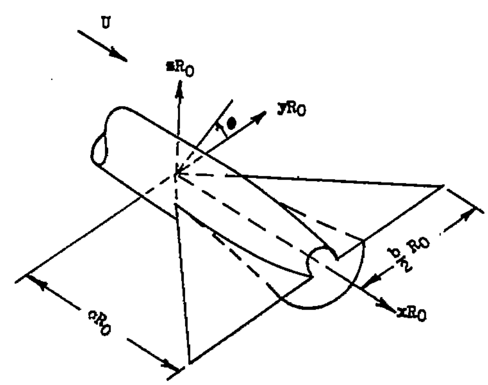
Superscripts:

b	refers to body
w	refers to wing

ANALYSIS

At supersonic speeds a lifting force on a wing-body configuration can be developed by contracting the body above the wing and expanding it below the wing. A configuration of this type which has the merit of simplicity of analysis consists of a flat-plate wing with supersonic edges and a body which (1) is cylindrical upstream of the wing and is at zero incidence, (2) is composed of semicircular parts of different radii above and below the wing, and (3) does not extend downstream of its intersection with the wing trailing edge. The simplicity of the analysis follows in part from the fact that the flow fields in the upper and lower half-spaces can be treated independently. The flow in each half-space can be represented by source distributions for the wing alone and body alone, together with an interference potential. With the body source distribution for one half-space the same as that for the other except for a change in sign, the body area at any station is equal to the area of the cylinder, and the pressure coefficients at corresponding points above and below the wing are numerically equal but opposite in sign. Thus, the lift or drag for the part of the configuration in either half-space is one-half the total.

The type of configuration considered herein is shown in sketch a. The unit of length is the cylinder radius R_O; the quantities x, y, z, $r = \sqrt{y^2 + z^2}$, the wing chord c, and the span b are nondimensional; and the wing is at an angle of attack α. Since the analysis is similar to that for other wing-body problems treated extensively in the literature, only a brief outline of the formulation is presented here and the details of the calculations are relegated to the appendix. The problem is formulated for the upper half-space.



Sketch a

The velocity potential Φ is taken as

$$\Phi = UR_O\left(x + \phi_w + \phi_b + \phi_i\right) \tag{1}$$

where the disturbance potentials satisfy the linearized differential equation for the velocity potential, ϕ_w and ϕ_b are the potentials for the wing alone and body alone, respectively, and ϕ_i is an interference potential required to satisfy the condition of zero normal velocity at the surface of the wing-body combination. The requirement of zero normal velocity on the wing is $\frac{\partial\Phi}{\partial z} = -\alpha \frac{\partial\Phi}{\partial x}$. With the definitions of the disturbance potentials together with the fact that $\partial\phi_b/\partial z$ is identically zero for $z = 0$ for a circular body, this boundary condition is approximated by

$$\frac{\partial\phi_w}{\partial z} = -\alpha \qquad \frac{\partial\phi_i}{\partial z} = 0 \qquad (z = 0) \qquad (2)$$

The requirement of zero normal velocity on the body is $\frac{\partial\Phi}{\partial x}\frac{dr_b}{dx} = \frac{\partial\Phi}{\partial r}$ at $r = r_b$, where r_b is the nondimensional body radius. With the restriction that the body does not depart substantially from the cylinder, this boundary condition is approximated by

$$\frac{\partial\phi_b}{\partial r} = \frac{dr_b}{dx} \qquad \frac{\partial\phi_w}{\partial r} + \frac{\partial\phi_i}{\partial r} = 0 \qquad (r = 1) \qquad (3)$$

The body potential is represented by a distribution of sources along the body axis, and in the linear approximation the pressure p_b due to the body is

$$\frac{p_b - p_\infty}{q} = \frac{1}{\pi}\int_0^{x-\beta r} \frac{f'(\xi)\, d\xi}{\sqrt{(x - \xi)^2 - \beta^2 r^2}} \qquad (4)$$

where the prime denotes the derivative with respect to the indicated argument, $\beta = \sqrt{M^2 - 1}$, and M is the stream Mach number. The source strength $f(\xi)$ is related to the body geometry by the first of equations (3). An equivalent statement of this boundary condition for quasi-cylindrical flows relates the body area in the upper half-space $A(x)$ to the source strength by

$$A(x) - \frac{\pi R_0^2}{2} \equiv \Delta A(x) = \frac{R_0^2}{2}\int_0^{x-\beta} f'(\xi)\sqrt{(x - \xi)^2 - \beta^2}\, d\xi \qquad (5)$$

where $\Delta A(x)$ is the change in area in the upper half-space. Since the body source strength for the flows in the upper and lower half-spaces differ only in sign, $\Delta A(x)$ for each half-space is the same except for a change in sign, and the total body area at any station is equal to the area of the cylinder $\pi R_0{}^2$.

The interference potential is required to cancel the flow through the body due to the wing potential and is represented by a distribution of multipoles along the body axis as

$$\phi_i = -\frac{1}{2\pi}\sum_{n=0}^{\infty}\begin{Bmatrix}\cos \\ \sin\end{Bmatrix} n\theta \int_0^{x-\beta r} \frac{h_n(\xi)\ \cosh\left(n\ \cosh^{-1}\frac{x - \xi}{\beta r}\right)\ d\xi}{\sqrt{(x - \xi)^2 - \beta^2 r^2}}$$

where $h_n(\xi)$ is the multipole strength and θ is the azimuth angle defined in sketch a. The boundary condition on ϕ_i as given by equation (3) is satisfied in each half-space by expanding $\frac{\partial\phi_w}{\partial r}$ in a Fourier series and equating the coefficients of each harmonic to the corresponding coefficient of $\frac{\partial\phi_i}{\partial r}$. Because of the symmetry properties of the wing, only the $\cos 2n\theta$ terms appear, and from equation (3)

$$-\frac{\partial\phi_w}{\partial r} = \frac{\partial\phi_i}{\partial r} = \alpha\sum_{n=0}^{\infty} g_{2n}(x)\ \cos 2n\theta \qquad (r = 1) \qquad (6)$$

where the Fourier coefficients are given by

$$\alpha\ g_0(x) = -\frac{1}{\pi}\int_0^{\pi} \left.\frac{\partial\phi_w}{\partial r}\right|_{r=1} d\theta$$

$$\alpha\ g_{2n}(x) = -\frac{2}{\pi}\int_0^{\pi} \left.\frac{\partial\phi_w}{\partial r}\right|_{r=1} \cos 2n\theta\ d\theta$$

The interference potential with $\phi_i \propto \cos 2n\theta$ satisfies the boundary condition as given by equation (2) for ϕ_i identically. The multipole strengths can be determined from equation (6) by replacing ϕ_i by the expression for a distribution of multipoles. However, this step is not necessary since only the pressure is required and Nielsen (ref. 2) has shown that the interference pressure for a quasi-cylindrical body can be related directly to the functions $g_{2n}(x)$ and a tabulated function $W_{2n}(x,r)$. This expression for the interference pressure p_i is

$$\frac{p_i - p_\infty}{q} = \frac{2\alpha}{\beta}\sum_{n=0}^{\infty} \cos 2n\theta\left[\frac{1}{\sqrt{r}}\, g_{2n}(x-\beta r+\beta) - \frac{1}{\beta}\int_{\beta}^{x-\beta(r-1)} g_{2n}(\xi)\, W_{2n}\left(\frac{x-\xi}{\beta} - r+1, r\right) d\xi\right] \tag{7}$$

The pressure p_w due to the wing is determined directly from the streamwise disturbance velocity of the wing alone.

The lift and drag are determined by integrating the surface pressures. The total lift L is

$$\frac{L}{2R_0{}^2} = -\iint\limits_{\substack{\text{Wing}\\\text{panels}}} \left(p_b^w + p_w^w + p_i^w\right) dA - \int_{\beta}^{c} dx \int_0^{\pi}\left(p_b^b + p_w^b + p_i^b\right)\sin\theta\, d\theta$$

$$= \frac{L^w}{2R_0{}^2} + \frac{L^b}{2R_0{}^2} \tag{8}$$

where the superscript on the pressure refers to the surface where the pressure acts, w denoting the wing and b the body. The superscript w (or b) on a force refers to the force obtained by integrating over the wing (or body). Similarly the total drag D is

$$\frac{D}{2R_0{}^2} = \frac{\alpha L^w}{2R_0{}^2} + \frac{1}{\pi}\int_\beta^c \Delta A'(x)\ dx \int_0^\pi \left(p_b^b + p_w^b + p_i^b\right) d\theta$$

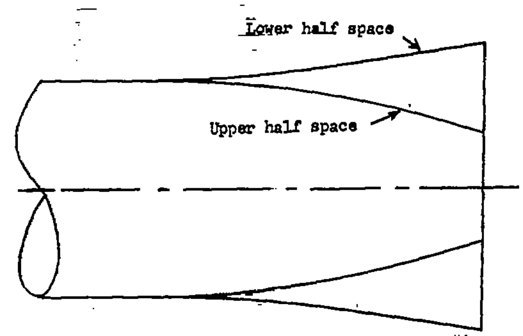
$$= \frac{D^w}{2R_0{}^2} + \frac{D^b}{2R_0{}^2} \tag{9}$$

where the first term is the drag of the wing panels and the second term is the body drag.

DISCUSSION

The method presented in the preceding section affords a means for evaluating the lift and drag characteristics of a class of wing-body configurations. For a given wing plan form, the forces can be determined most easily by specifying the body source distribution rather than the body geometry. Calculations have been made for a Mach number of $\sqrt{2}$ for a delta wing with sonic leading edges and a linear body source distribution. This configuration was selected primarily to simplify the calculations. However, it is believed that the results are generally indicative of the effect of a body cross-sectional shape on the lifting efficiency of a wing-body configuration at supersonic speeds.

The body area distribution is determined from equation (5) for the body source distribution given by $f(x) = -Bx$, where B is a positive constant. The body shape is shown in sketch b for the rather large value of $B = 0.35$. The contributions to the lift and drag from the disturbances arising from the wing and body can be integrated directly and are given in the appendix, whereas the contribution to the forces from the interference pressure must be determined numerically. The interference pressure depends only upon the geometric properties of the wing through the Fourier coefficients $g_{2n}(x)$. These functions have been evaluated for $n = 0$ and 1 in reference 2 and only the first two terms of the interference pressure have been used in the calculations. Although this limitation affects the accuracy of the calculations, Nielsen indicates that the major effect is contained in the first two terms.



Sketch b

The lift and drag coefficients, based on wing area, can be written as the sums

$$C_L = C_{L,w}^{\ w} + C_{L,b}^{\ w} + C_{L,i}^{\ w} + C_{L,w}^{\ b} + C_{L,b}^{\ b} + C_{L,i}^{\ b}$$

$$C_D = \alpha\left(C_{L,w}^{\ w} + C_{L,b}^{\ w} + C_{L,i}^{\ w}\right) + C_{D,w}^{\ b} + C_{D,b}^{\ b} + C_{D,i}^{\ b}$$

where the subscripts denote the origin of the pressure disturbance and the superscripts denote the surface upon which the pressures act. The lift coefficients with the subscripts w and i are proportional to α and those with subscript b are proportional to B, while $C_{D,w}^{\ b}$ and $C_{D,i}^{\ b}$ are proportional to $B\alpha$ and $C_{D,b}^{\ b}$ is proportional to B^2. Consequently, the ratio C_D/C_L^2, which is indicative of the lifting efficiency of the configuration, is a function only of B/α and c. The values of B/α which correspond to the minimum values of C_D/C_L^2 are denoted by $(B/\alpha)_{opt}$ and are shown in figure 1 as a function of c. In figure 2, values of C_D/C_L^2 are presented for the configuration with $(B/\alpha)_{opt}$ together with those for the wing alone, the wing-cylinder $(B = 0)$, and the indented cylinder with wing at zero angle of attack $(\alpha = 0)$. The curve for the wing with the optimum indented cylinder is the locus of points for body shapes which give rise to the least values of C_D/C_L^2; thus the body shape varies with angle of attack and c in accordance with figure 1. The curve for the wing alone, which exhibits the highest lifting efficiency, is presented primarily for the purpose of orientation - the most meaningful comparisons are those between the configurations which have a body.

Comparison of the curves for the wing-cylinder $(B = 0)$ and the indented cylinder with wing at $\alpha = 0$ shows that for large wings (that is, large values of c) lift is generated more efficiently by the wing-cylinder, whereas for small wings lift is generated more efficiently by shaping the body. Limiting calculations for large values of c show that the lifting efficiency of the wing-cylinder approaches that of the wing alone, while for the indented cylinder C_D/C_L^2 approaches infinity as c approaches infinity. The lifting efficiency that can be obtained by generating lift both directly and indirectly represents a sizable gain over the efficiency obtained with either method individually. The decrease of $(B/\alpha)_{opt}$ with increasing c reflects the fact that, for the optimum condition, proportionately more of the lift is developed by the system which itself is most efficient.

The quantities C_L/α and C_D/α^2 for the wing, the wing-cylinder, and the configuration with $(B/\alpha)_{opt}$ are presented in figure 3 as functions of c. The value of C_L/α for the configuration with $(B/\alpha)_{opt}$ is substantially greater than for the wing-cylinder and is nearly equal to that of the wing alone for c equal to 5. The quantity C_D/α^2 for the configuration with $(B/\alpha)_{opt}$ is approximately equal to that for the wing alone for $c = 2$, and becomes approximately 10 percent higher at the larger values of c. The value of C_D/α^2 for the wing-cylinder is substantially lower since the cylindrical body does not contribute to the drag. The differences between the curves of figure 3 for the configuration with $(B/\alpha)_{opt}$ and the wing-cylinder represents the contribution to the forces due to shaping the body.

The contributions to the lift and drag coefficients are given in figures 4 and 5 for the optimum configuration. The contributions to the lift and drag from the interference pressure produces a down load and thrust. It is of interest to note that, for the optimum condition, the lift coefficient is closely equal to $C_{L,w}^{\ w} + C_{L,b}^{\ b}$ since the terms $C_{L,i}^{\ w}$ and $C_{L,i}^{\ b}$ nearly cancel $C_{L,b}^{\ w}$ and $C_{L,w}^{\ b}$, respectively. Similarly, for the optimum condition, the terms $C_{D,b}^{\ w}$ and $C_{D,i}^{\ w}$ nearly cancel while $C_{D,w}^{\ b}$ and $C_{D,i}^{\ b}$ together contribute a small value which is nearly independent of c. The terms $C_{L,b}^{\ b}$ and $C_{D,b}^{\ b}$ decrease in value with increasing c, reflecting the fact that for the optimum condition B/α decreases as the ratio of wing span to cylinder radius increases.

The effectiveness of using indirect lift to increase the lifting efficiency of wing-body combinations is dependent to a large extent on the body configuration. Although only the portion of the body in the neighborhood of the wing contributes to the indirect lift, the shape ahead of the wing affects the body drag and consequently the optimum ratio of direct to indirect lift. Nevertheless, the example presented serves to illustrate the effect of body cross-sectional shape on the lifting efficiency. The family of body shapes used in the example was selected for simplicity of analysis. It seems clear, however, that other area distributions would be superior for developing indirect lift. For example, the body shape in the neighborhood of the wing (for each half-space) corresponding to a square-root type of source distribution, a type which arises in various minimum-drag problems for bodies, would probably lead to a lifting system more efficient than that of the example since it would be favorable to both the body drag and the wing lift.

The effectiveness of developing lift by utilizing the pressure field from the body is dependent upon the wing plan form and upon the stream Mach number. The increase in lifting efficiency obtained by utilizing indirect lift might be expected to be somewhat higher than that of the example for wing plan forms, such as swept wings, which could make use of more of the pressure field arising from the body to develop lift. Moreover, it would appear that body shaping is most effective for the condition where the wing leading edges are sonic, since the pressure relief for wings with subsonic edges and the smaller wing area affected by the pressure field from the body for wings with supersonic edges would result in less lift from the interfering body elements.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 18, 1956.

APPENDIX

EVALUATION OF THE PRESSURES AND FORCES FOR THE EXAMPLE

The expressions for the pressures and forces for a delta wing on a quasi-cylindrical body are presented herein. The body shape for each half-space corresponds to that given by a linear source distribution. The calculations are for the sonic edge condition and a Mach number of $\sqrt{2}$.

From equations (8) and (9) the lift and drag coefficients for this configuration are

$$\left.\begin{aligned}
C_L &= -\frac{4}{qc^2}\int_1^c dy\int_y^c \left(p_b^w + p_w^w + p_i^w\right) dx - \frac{2}{qc^2}\int_1^c dx\int_0^\pi \left(p_b^b + p_w^b + \bar{p}_i^b\right)\sin\theta\, d\theta \\
C_D &= -\frac{4\alpha}{qc^2}\int_1^c dy\int_y^c \left(p_b^w + p_w^w + p_i^w\right) dx + \frac{2}{q\pi c^2}\int_1^c \Delta A'(x)\, dx\int_0^\pi \left(\bar{p}_b^b + p_w^b + p_i^b\right) d\theta
\end{aligned}\right\} \tag{A1}$$

For a linear source strength $f(\xi) = -B\xi$, where B is a positive constant, the body shape in the upper half-space is determined from equation (5) as

$$\Delta A(x) = -\frac{B}{2} R_0{}^2\left(x\sqrt{x^2 - 1} - \cosh^{-1} x\right) \tag{A2}$$

The pressure due to the body is determined from equation (4) as

$$\frac{p_b - p_\infty}{q} = -\frac{B}{\pi}\cosh^{-1}\frac{x}{r} \tag{A3}$$

and p_b^b and p_b^w are given by equation (A3) for $r = 1$ and $r = y$, respectively.

The line pressure sources of reference 3 can be used to determine the pressure due to the wing. For the sonic edge condition and $\beta = 1$, p_W becomes

$$\frac{p_W - p_\infty}{q} = -\frac{4\alpha}{\pi}\frac{x}{r}\frac{\sqrt{\left(\frac{x}{r}\right)^2 - 1}}{\left(\frac{x}{r}\right)^2 - \cos^2\theta} \tag{A4}$$

From equation (A4), p_W^b is obtained for $r = 1$ and p_W^W is obtained for $\theta = 0$ and $r = y$.

The interference pressure must be evaluated numerically since it involves the tabulated functions $g_{2n}(x)$ and W_{2n}. These functions have been evaluated for a flat-plate wing in reference 1 for $n = 0$ and $n = 1$. The functions $g_{2n}(x)$ used herein correspond to the functions $-f_{2ns}(x)$ given in figure 2 of the reference; the function $g_0(x)$ is a constant equal to $2/\pi$. The interference pressure on the wing p_i^W is determined from equation (7) by retaining only the first two terms of the infinite series for p_i, and setting z equal to 0 and θ equal to 0:

$$\frac{p_i^W - p_\infty}{q} = 2\alpha\sum_{n=0}^{1}\left[\frac{1}{\sqrt{y}}\, g_{2n}(x-y+1) - \int_1^{x-y+1} g_{2n}(\xi)\, W_{2n}(x-\xi-y+1,y)\, d\xi\right] \tag{A5}$$

The pressure on the body p_i^b is given for $r = 1$ as

$$\frac{p_i^b - p_\infty}{q} = 2\alpha\sum_{n=0}^{1}\cos 2n\theta\left[g_{2n}(x) - \int_1^{x} g_{2n}(\xi)\, W_{2n}(x-\xi,1)\, d\xi\right] \tag{A6}$$

When the expressions for the body shape and pressures are substituted into equations (A1), the various contributions to the lift and drag of the wing are

$$\frac{C_{L,b}^{\;w}}{\alpha} = \frac{C_{D,b}^{\;w}}{\alpha^2} = \frac{-4}{q\alpha c^2}\int_1^c dy \int_y^c p_b^w \, dx = \frac{2}{\pi c^2}\frac{B}{\alpha}\left(c^2 \cos^{-1}\frac{1}{c} - 2c \cosh^{-1}c + \sqrt{c^2 - 1}\right)$$

$$\frac{C_{L,w}^{\;w}}{\alpha} = \frac{C_{D,w}^{\;w}}{\alpha^2} = \frac{-4}{q\alpha c^2}\int_1^c dy \int_y^c p_w^w \, dx = \frac{8}{\pi c^2}\left(c^2 \cos^{-1}\frac{1}{c} - \sqrt{c^2 - 1}\right)$$

$$\frac{C_{L,i}^{\;w}}{\alpha} = \frac{C_{D,i}^{\;w}}{\alpha^2} = \frac{-4}{q\pi c^2}\int_1^c dy \int_y^c p_i^w \, dx$$

and the contributions to the lift and drag of the body are

$$\frac{C_{L,b}^{\;b}}{\alpha} = -\frac{2}{q\alpha c^2}\int_1^c dx \int_0^\pi p_b^b \sin\theta \, d\theta = \frac{4B}{\pi c^2 \alpha}\left(c \cosh^{-1} c - \sqrt{c^2 - 1}\right)$$

$$\frac{C_{L,w}^{\;b}}{\alpha} = -\frac{2}{q\alpha c^2}\int_1^c dx \int_0^\pi p_w^b \sin\theta \, d\theta$$

$$= \frac{8}{\pi c^2}\left[\coth^{-1} c\left(c\sqrt{c^2 - 1} - \cosh^{-1} c\right) + \sqrt{c^2 - 1} - I\left(\cosh^{-1} c\right)\right]$$

where

$$I(\xi) = \xi - \frac{\xi^3}{3 \cdot 3!} + \frac{7\xi^5}{3 \cdot 5 \cdot 5!} - \frac{31\xi^7}{3 \cdot 7 \cdot 7!} + \frac{127\xi^9}{3 \cdot 5 \cdot 9!} - \ . \ . \ .$$

$$\frac{C_{L,i}^{\ b}}{\alpha} = - \frac{2}{q\alpha c^2} \int_1^c dx \int_0^\pi p_i^b \sin\theta \, d\theta$$

$$\frac{C_{D,b}^{\ b}}{\alpha^2} = \frac{1}{q\pi c^2 \alpha^2} \int_1^c \Delta A'(x) \, dx \int_0^\pi p_b^b \, d\theta$$

$$= \frac{1}{4\pi c^2}\left(\frac{B}{\alpha}\right)^2\left[\cosh^{-1} c\left(2c\sqrt{c^2 - 1} - \cosh^{-1} c\right) + 1 - c^2\right]$$

$$\frac{C_{D,w}^{\ b}}{\alpha^2} = \frac{1}{q\pi c^2 \alpha^2} \int_1^c \Delta A'(x) \, dx \int_0^\pi p_w^b \, d\theta$$

$$= \frac{1}{\pi c^2} \frac{B}{\alpha}\left(c\sqrt{c^2 - 1} - \cosh^{-1} c\right)$$

$$\frac{C_{D,i}^{\ b}}{\alpha^2} = \frac{1}{q\pi c^2 \alpha^2} \int_1^c \Delta A'(x) \, dx \int_0^\pi p_i^b \, d\theta$$

$$= - \frac{4}{\pi c^2} \frac{B}{\alpha}\left[\frac{1}{2}\left(c\sqrt{c^2 - 1} - \cosh^{-1} c\right) - \int_1^c \sqrt{x^2 - 1} \, dx \int_1^x W_0(x-\xi,1) \, d\xi\right]$$

The coefficients $\frac{C_{L,i}^{\ w}}{\alpha}$ $\left(\text{or } \frac{C_{D,i}^{\ w}}{\alpha^2}\right)$, $\frac{C_{L,i}^{\ b}}{\alpha}$, and $\frac{C_{D,i}^{\ b}}{\alpha^2}$ have been integrated numerically by using the first two terms of the series representation. The various contributions to the lift and drag are presented in figures 4 and 5 as a function of c f[redacted]f B/α which minimize C_D/C_L^2.

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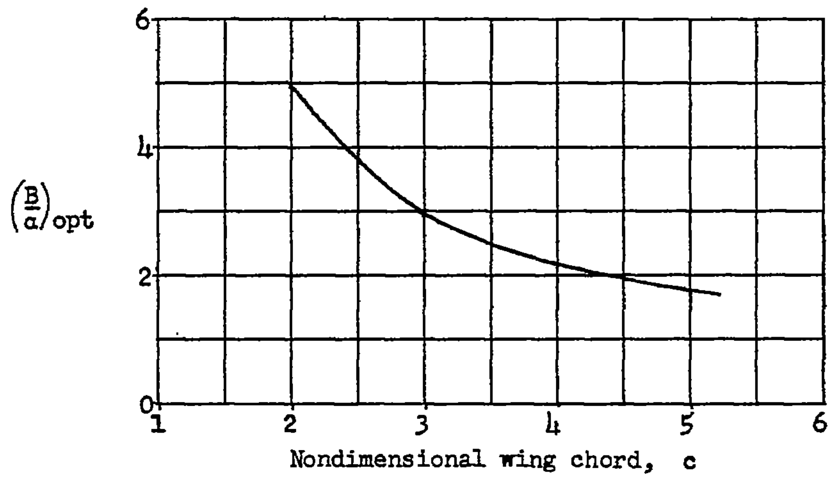

Figure 1.- Variation of B/α with c for minimum value of $C_D/C_L{}^2$.

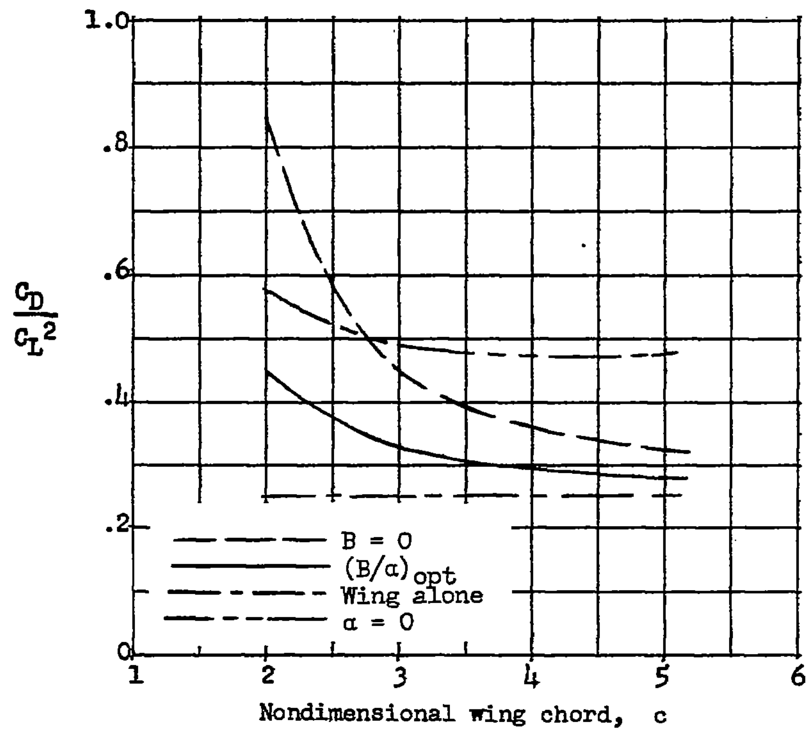

Figure 2.- Variation of $C_D/C_L{}^2$ with c for various configurations.

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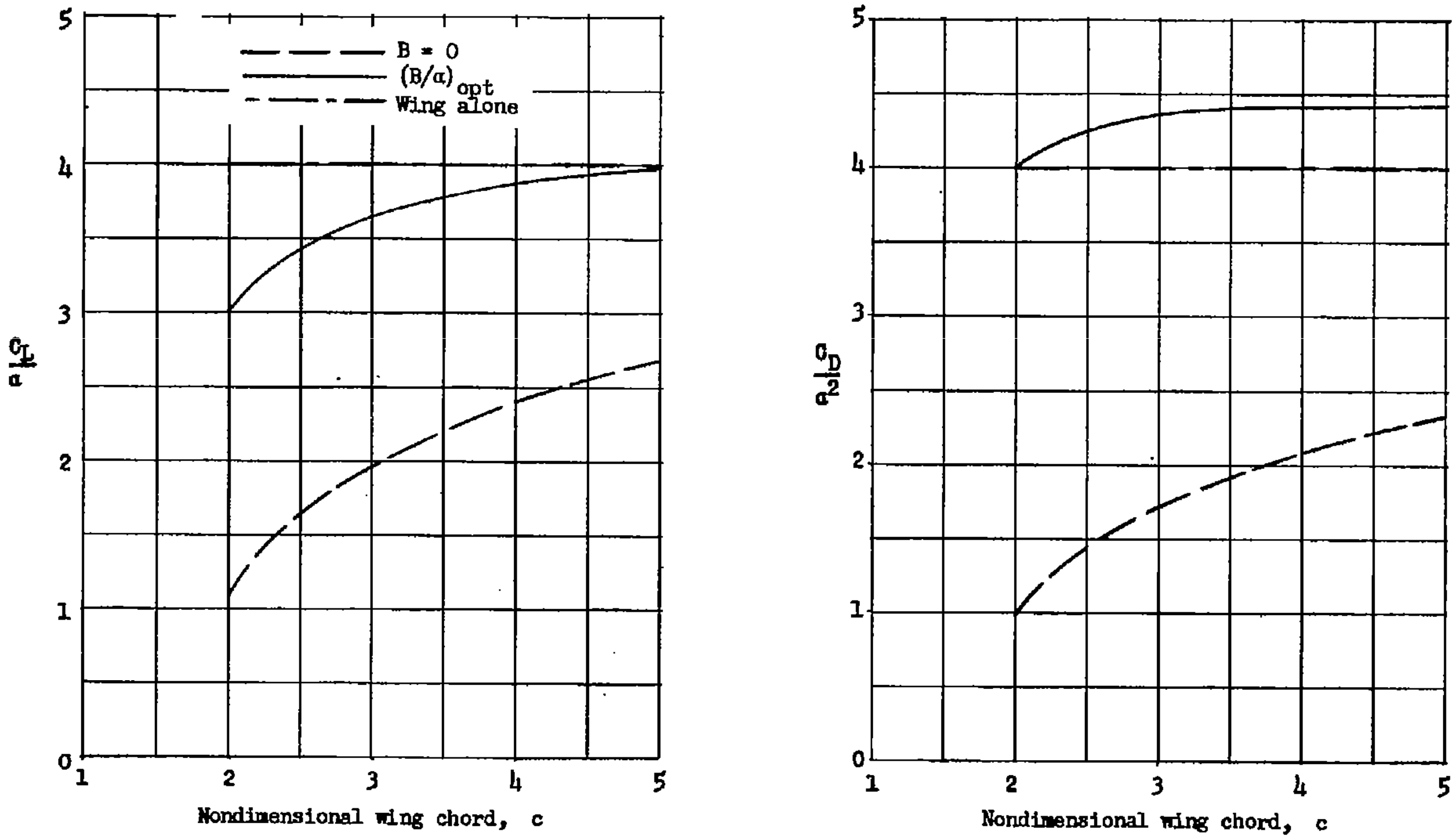

Figure 3.- Variation of C_L/α and C_D/α^2 with c for various configurations.

[redacted]

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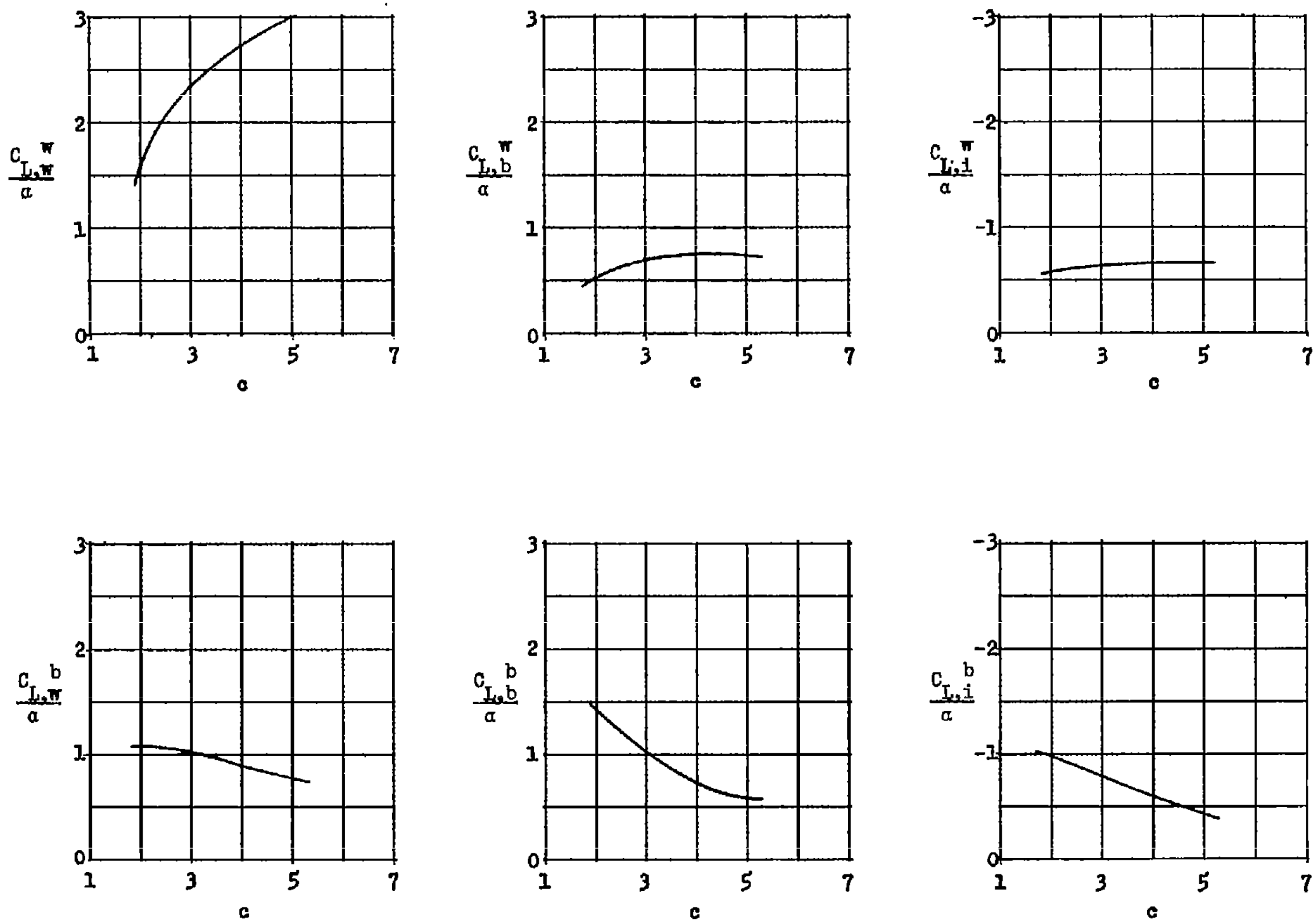

Figure 4.- Component terms of C_L/α as a function of the nondimensional wing chord c for the optimum configurations.

[redacted]

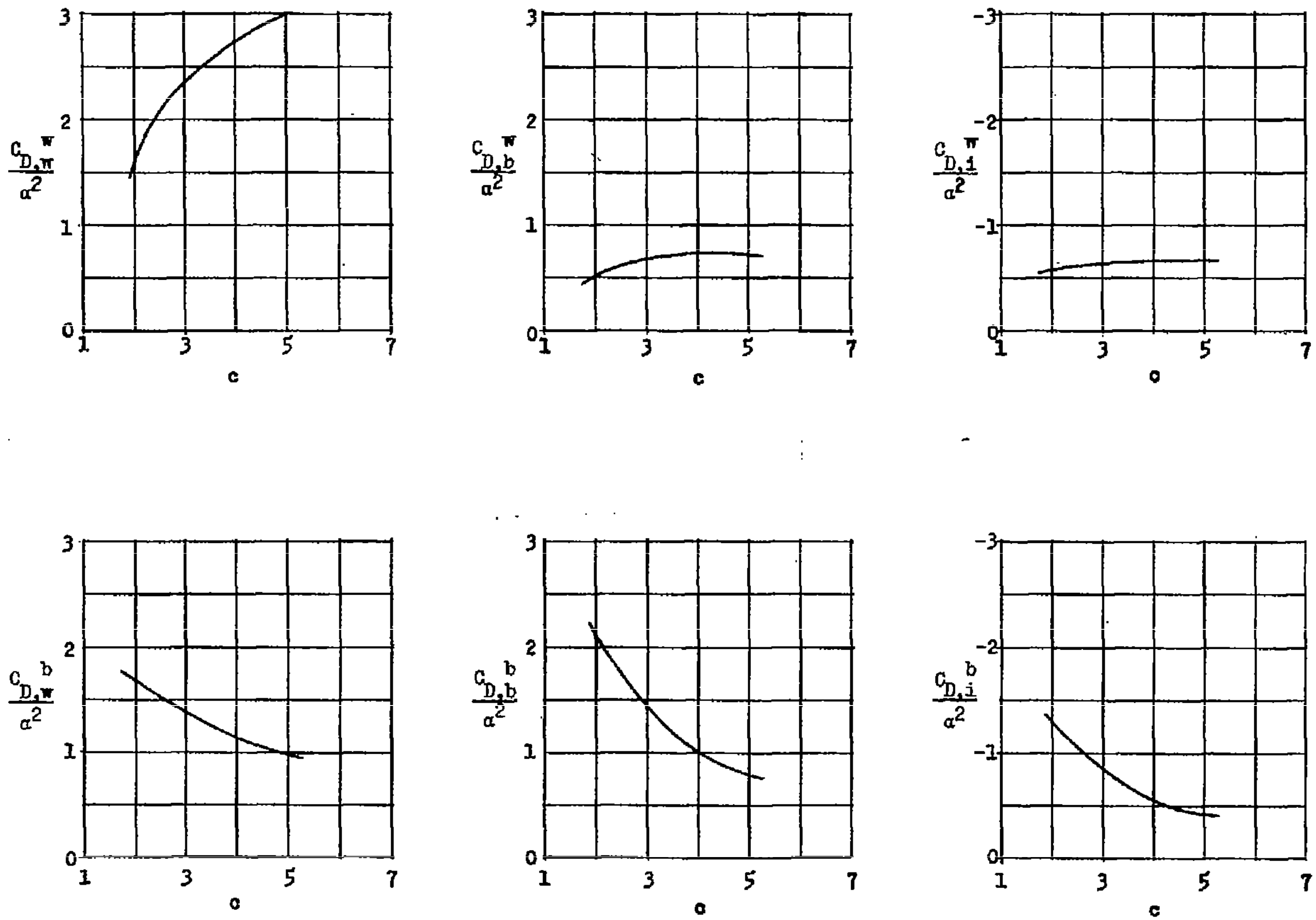

Figure 5.- Component terms of C_D/α^2 as a function of the nondimensional wing chord c for the optimum configurations.

[redacted]